應用力學下　　　　　Quiz 1　　命題老師：廖國基教授　　　　10/2022

**注意：**

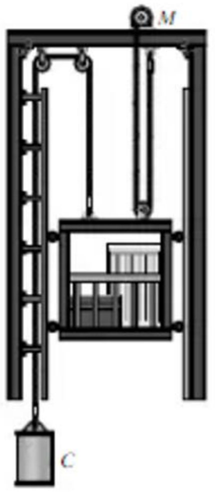
**1.　所有題目均需詳列計過程，否則不予計分。**

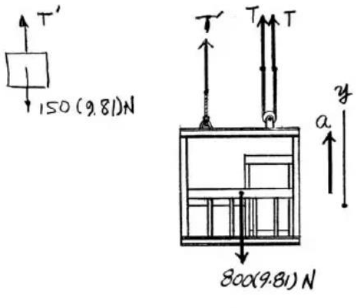
**2.　除計算機外，禁止使用如書籍、筆記、講義等任何形式之輔助工具。**

**3.　請於試卷謄寫下列之 Honor Code。**

**"本人於此次考試中沒有接受任何形式之外部協助作答"(簽名)**

1. The material hoist and the load have a total mass of 800 kg and the counterweight *C* has a mass of 150 kg. If the upward speed of the hoist increases uniformly from 0.5 m/s to 1.5 m/s in 1.5 s, determine the average power generated by the motor *M* during this time. The motor operates with an efficiency of $\epsilon = 0.8$.




(a)

**Kinematics:** The acceleration of the hoist can be determined from

$(+\uparrow)\qquad v = v_0 + a_c t$

$$1.5 = 0.5 + a(1.5)$$

$$a = 0.6667 \text{ m/s}^2$$

**Equations of Motion:** Using the result of **a** and referring to the free-body diagram of the hoist and block shown in Fig. *a*,

$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(0.6667)$

$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T' = 150(0.6667)$

Solving,

$$T' = 1371.5 \text{ N}$$

$$T = 3504.92 \text{ N}$$

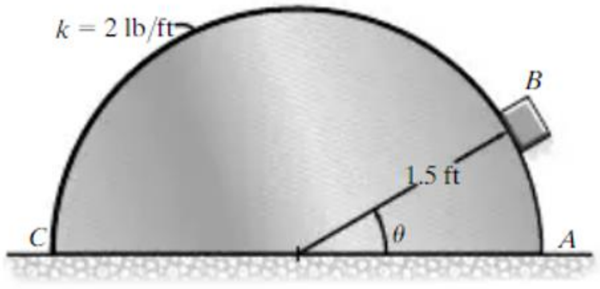
**Power:**

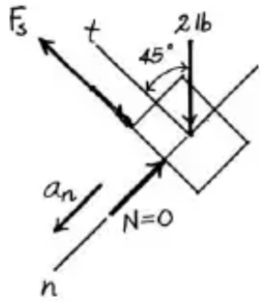
$$(P_{out})_{avg} = 2\mathbf{T} \cdot \mathbf{v}_{avg} = 2(3504.92)\left(\frac{1.5 + 0.5}{2}\right) = 7009.8 \text{ W}$$

Thus,

$$P_{in} = \frac{P_{out}}{\varepsilon} = \frac{7009.8}{0.8} = 8762.3 \quad \text{W} = 8.76 \text{ kW} \qquad \textbf{Ans.}$$

2. A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at $B$ and to the base of the semicylinder at point $C$. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so that the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.




$+\swarrow \Sigma F_n = ma_n;\qquad 2\sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$

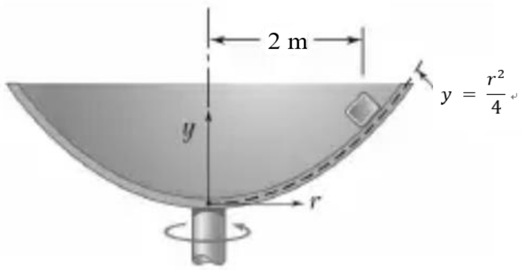
$v = 5.844$ ft/s

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + \frac{1}{2}(2)\left[\pi(1.5) - l_0\right]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5\sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$$

$l_0 = 2.77$ ft **Ans.**

3. A 3-kg block is at rest relative to a parabolic dish which rotates at a constant rate about a vertical axis. Knowing that the coefficient of static friction is 0.5 and that $r$ = 2 m, determine the maximum allowable velocity $v$ of the block.



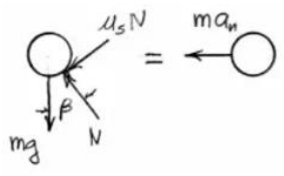

Let $\beta$ be the slope angle of the dish. $\tan\beta = \frac{dy}{dr} = \frac{1}{2}r$

At $r = 2$ m, $\tan\beta = 1$ or $\beta = 45°$

Draw free body sketches of the sphere.

$$\Sigma F_y = 0: \quad N\cos\beta - \mu_S N\sin\beta - mg = 0$$

$$N = \frac{mg}{\cos\beta - \mu_S \sin\beta}$$



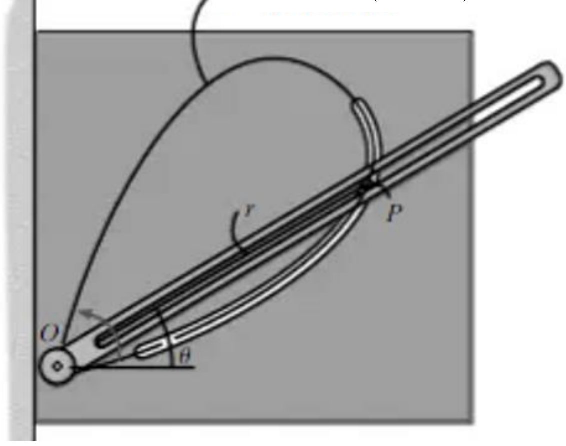

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n: \quad N\sin\beta + \mu_S N\cos\beta = \frac{mv^2}{\rho}$$

$$\frac{mg(\sin\beta + \mu_S N\cos\beta)}{\cos\beta - \mu_S\sin\beta} = \frac{mv^2}{\rho}$$

$$v^2 = \rho g \frac{\sin\beta + \mu_S\cos\beta}{\cos\beta - \mu_S\sin\beta} = (2)(9.81)\frac{\sin 45° + 0.5\cos 45°}{\cos 45° - 0.5\sin 45°} = 58.86 \text{ m}^2/\text{s}^2$$

$v = 7.67$ m/s ◀

4. If arm $OA$ rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 2$ rad/s, determine the magnitudes of the velocity and acceleration of peg $P$ at $\theta = 30^\circ$. The peg moves in the fixed groove defined by the lemniscate, and along the slot in the arm.




**Time Derivatives:**

$$r^2 = 4\sin 2\theta$$

$$2r\dot{r} = 8\cos 2\theta\dot{\theta}$$

$$\dot{r} = \left[\frac{4\cos 2\theta\dot{\theta}}{r}\right] \text{m/s} \qquad \dot{\theta} = 2 \text{ rad/s}$$

$$2\left(r\ddot{r} + \dot{r}^2\right) = 8\left(-2\sin 2\theta\dot{\theta}^2 + \cos 2\theta\ddot{\theta}\right)$$

$$\ddot{r} = \left[\frac{4\left(\cos 2\theta\ddot{\theta} - 2\sin 2\theta\dot{\theta}^2\right) - \dot{r}^2}{r}\right] \text{m/s}^2 \qquad \ddot{\theta} = 0$$

At $\theta = 30^\circ$,

$$r\big|_{\theta=30^\circ} = \sqrt{4\sin 60^\circ} = 1.861 \text{ m}$$

$$\dot{r}\big|_{\theta=30^\circ} = \frac{(4\cos 60^\circ)(2)}{1.861} = 2.149 \text{ m/s}$$

$$\ddot{r}\big|_{\theta=30^\circ} = \frac{4\left[0 - 2\sin 60^\circ\left(2^2\right)\right] - (2.149)^2}{1.861} = -17.37 \text{ m/s}^2$$

**Velocity:**

$$v_r = \dot{r} = 2.149 \text{ m/s} \qquad v_\theta = r\dot{\theta} = 1.861(2) = 3.722 \text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{2.149^2 + 3.722^2} = 4.30 \text{ m/s} \qquad \textbf{Ans.}$$

**Acceleration:**

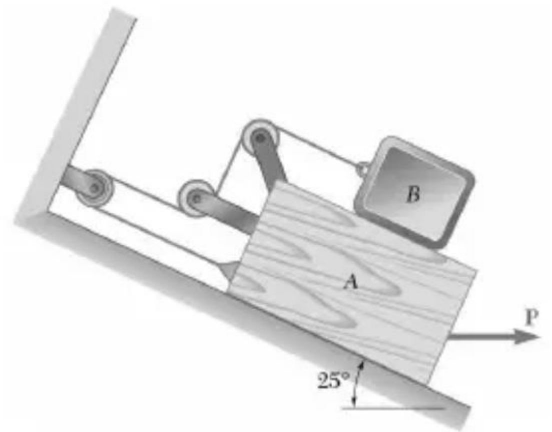
$$a_r = \ddot{r} - r\dot{\theta}^2 = -17.37 - 1.861\left(2^2\right) = -24.82 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(2.149)(2) = 8.597 \text{ m/s}^2$$

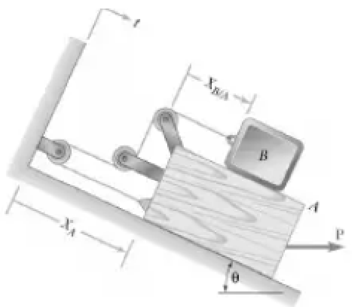
Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-24.82)^2 + 8.597^2} = 26.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

5. Block $A$ has a mass of 40 kg, and block $B$ has a mass of 8 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ and $\mu_k = 0.15$. If $P = 0$, determine (a) the acceleration of block $B$. (b) the tension in the cord.




From the constraint of the cord:




$$2x_A + x_{B/A} = \text{constant}$$

Then $$2v_A + v_{B/A} = 0$$

and $$2a_A + a_{B/A} = 0$$

Now $$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Then $$a_B = a_A + (-2a_A)$$

or $$a_B = -a_A \qquad (1)$$

First we determine if the blocks will move for the given value of $\theta$. Thus, we seek the value of $\theta$ for which the blocks are in impending motion, with the impending motion of $A$ down the incline.

$B$:

$B$: $+\nearrow \Sigma F_y = 0$: $\quad N_{AB} - W_B \cos\theta = 0$

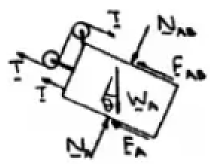
or $$N_{AB} = m_B g \cos\theta$$

Now $$F_{AB} = \mu_s N_{AB}$$
$$= 0.2 m_B g \cos\theta$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T + F_{AB} + W_B \sin\theta = 0$$

or $$T = m_B g(0.2\cos\theta + \sin\theta)$$

$A$:



$A$: $+\nearrow \Sigma F_y = 0$: $\quad N_A - N_{AB} - W_A \cos\theta = 0$

or $$N_A = (m_A + m_B) g \cos\theta$$

Now $$F_A = \mu_s N_A$$
$$= 0.2(m_A + m_B) g \cos\theta$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T - F_A - F_{AB} + W_A \sin\theta = 0$$

or $$T = m_A g \sin\theta - 0.2(m_A + m_B)g\cos\theta - 0.2 m_B g\cos\theta$$
$$= g[m_A \sin\theta - 0.2(m_A + 2m_B)\cos\theta]$$

Equating the two expressions for $T$

$$m_B g(0.2\cos\theta + \sin\theta) = g[m_A \sin\theta - 0.2(m_A + 2m_B)\cos\theta]$$

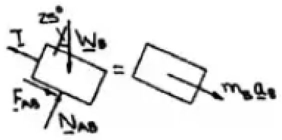
or $$8(0.2 + \tan\theta) = [40\tan\theta - 0.2(40 + 2\times 8)]$$

or $$\tan\theta = 0.4$$

or $\theta = 21.8°$ for impending motion. Since $\theta < 25°$, the blocks will move. Now consider the motion of the blocks.

(*a*) $+\nearrow\ \Sigma F_y = 0$: $N_{AB} - W_B \cos 25° = 0$

B:



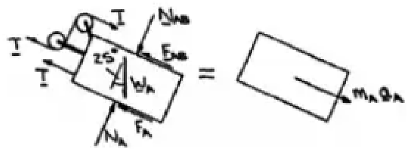
or $$N_{AB} = m_B g\cos 25°$$

Sliding: $$F_{AB} = \mu_k N_{AB} = 0.15 m_B g\cos 25°$$

$$\overset{+}{\searrow}\Sigma F_x = m_B a_B:\quad -T + F_{AB} + W_B \sin 25° = m_B a_B$$

or $$T = m_B[g(0.15\cos 25° + \sin 25°) - a_B]$$
$$= 8[9.81(0.15\cos 25° + \sin 25°) - a_B]$$
$$= 8(5.47952 - a_B) \qquad \text{(N)}$$

A:



$$+\nearrow\ \Sigma F_y = 0:\quad N_A - N_{AB} - W_A\cos 25° = 0$$

or $$N_A = (m_A + m_B)g\cos 25°$$

Sliding: $$F_A = \mu_k N_A = 0.15(m_A + m_B)g\cos 25°$$

$$\overset{+}{\searrow}\Sigma F_x = m_A a_A:\quad -T - F_A - F_{AB} + W_A \sin 25° = m_A a_A$$

Substituting and using Eq. (1)

$$T = m_A g \sin 25° - 0.15(m_A + m_B)g\cos 25°$$
$$-0.15 m_B g\cos 25° - m_A(-a_B)$$
$$= g[m_A \sin 25° - 0.15(m_A + 2m_B)\cos 25°] + m_A a_B$$
$$= 9.81[40\sin 25° - 0.15(40 + 2\times 8)\cos 25°] + 40a_B$$
$$= 91.15202 + 40a_B \qquad \text{(N)}$$

Equating the two expressions for $T$

$$8(5.47952 - a_B) = 91.15202 + 40a_B$$

or $$a_B = -0.98575 \text{ m/s}^2$$

$$\mathbf{a}_B = 0.986 \text{ m/s}^2 \searrow 25° \blacktriangleleft$$

(*b*) We have $$T = 8[5.47952 - (-0.98575)]$$

or $$T = 51.7 \text{ N} \blacktriangleleft$$