## 注意：

1．所有題目均需詳列計過程，否則不予計分。
2．除計算機外，禁止使用如書籍，筆記，講義等任何形式之輔助工具。
3．請於試卷謄䔍下列之 Honor Code 。

## ＂本人於此次考試中沒有接受任何形式之外部協助作答＂（簽名）

1．Using a slingshot，the boy fires the $0.2-\mathrm{kg}$ marble at the concrete wall，striking it at $B$ ．If the coefficient of restitution between the marble and the wall is $e=0.5$ ，determine the speed of the marble after it rebounds from the wall．

$(\stackrel{+}{\rightarrow})\left(S_{B}\right)_{x}=\left(S_{A}\right)_{x}+\left(v_{A}\right)_{x} t$

$$
\begin{aligned}
30 & =0+22.5 \cos 45^{\circ} t \\
t & =1.886 \mathrm{~s}
\end{aligned}
$$

$$
\begin{aligned}
(+\uparrow)\left(S_{B}\right)_{y} & =\left(S_{A}\right)_{y}+\left(V_{A}\right)_{y} t+\frac{1}{2} \alpha_{y} t^{2} \\
\left(S_{B}\right)_{y} & =0+22.5 \sin 45^{\circ}(1.886)+\frac{1}{2}(-9.8)(1.886)^{2} \\
& =12.57 \mathrm{~m}
\end{aligned}
$$



$$
\begin{aligned}
(+\uparrow)\left(v_{B}\right)_{y} & =\left(v_{A}\right)_{y}+a_{y} t \\
& =22.5 \sin 45^{\circ}+(-9.8)(1.886)=2.573 \mathrm{~m} / \mathrm{s} \\
\left(v_{B}\right)_{x} & =\left(v_{A}\right)_{x}=22.5 \cos 45^{\circ}=15.910^{\mathrm{m} / \mathrm{s}} \\
v_{B} & =\sqrt{(\sqrt{B})_{x}^{2}+(\sqrt{B})_{y}^{2}}=16.117 \mathrm{~m} / \mathrm{s} \\
\theta & =\tan ^{-1}\left[\frac{(\sqrt{B})_{y}}{(\sqrt{B})_{x}}\right]=9.17^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
&(+1) \quad m_{B}\left(v_{B}\right) x_{x^{\prime}}=m_{B}\left(v_{B}^{\prime}\right)_{x^{\prime}} \\
& 0.2[16.117 \sin (30-9.17)]=0.2\left(v_{B}^{\prime} \cos \phi\right) \\
& v_{B}^{\prime} \cos \phi=5.731 \mathrm{~m} / \mathrm{s} \\
&(+\uparrow) \quad e=\frac{0-\left(v_{B}^{\prime}\right) y^{\prime}}{\left(v_{B}\right) y_{y^{\prime}}-0} \\
& 0.5=\frac{-v_{B}^{\prime} \sin \phi}{-16.117 \cos (30-9.17)} \\
& v_{B}^{\prime} \sin \phi=7.531 \mathrm{~m} / \mathrm{s} \\
& v_{B}^{\prime}=\sqrt{(5.731)^{2}+(7.531)^{2}}=9.463 \mathrm{~m} / \mathrm{s} \\
& \frac{1}{2} m v_{B}^{2}+m g \mathrm{v}_{B}=\frac{1}{2} \mathrm{~m} v_{c}^{2}+m g \mathrm{~h}_{c}^{\prime} \\
& \frac{1}{2} m(9.463)^{2}+m(9.8)(1.5+12.57)=\frac{1}{2} \mathrm{~m} v_{c}^{2}
\end{aligned}
$$

$$
v_{c}=19.113 \mathrm{~m} / \mathrm{s} \text { \# Ans. }
$$

2. The $80-\mathrm{mm}$-radius wheel shown rolls to the left with a velocity of $900 \mathrm{~mm} / \mathrm{s}$. Knowing that the distance $A D$ is 50 mm , determine the velocity of the collar and the angular velocity of rod $A B$ when (a) $\beta=0$, (b) $\beta=90^{\circ}$.

(a) $\beta=0$.

(b) $\beta=90^{\circ}$.


Wheel $\left.A D . \quad \mathbf{v}_{C}=0, \quad \omega_{A D}=11.25 \mathrm{rad} / \mathrm{s}\right)$
Wheel $A D . \quad \mathbf{v}_{C}=0, \quad \mathbf{v}_{D}=45 \mathrm{in} . \mathrm{k}$,
$\left.\omega_{A D}=\frac{v_{D}}{C D}=\frac{900}{80}=11.25 \mathrm{rad} / \mathrm{s}\right)$
$C A=(C D)-(D A)=80-50=30 \mathrm{~mm}$
$v_{A}=(C A) \omega_{A D}=(30)(11.25)=337.5 \mathrm{~mm} / \mathrm{s} \leftarrow$
$\operatorname{Rod} A B . \quad \mathbf{v}_{B}=\mathbf{v}_{A}+\mathbf{v}_{B / A}$
$\left[v_{B} \longleftarrow\right]=[337.5 \longleftarrow]+\left[v_{B / A} \forall \varphi\right] \quad \mathbf{v}_{B}=338 \mathrm{~mm} / \mathrm{s} \longleftarrow<$
$v_{B / A}=0$ $\omega_{A B}=0$

$$
\tan \gamma=\frac{D A}{D C}=\frac{50}{80}, \quad \gamma=32.005^{\circ}
$$

$$
C A=\frac{D C}{\cos \gamma}=94.34 \mathrm{~mm}
$$



$$
v_{A}=(C A) \omega_{A D}=(94.34)(11.25)=1061.3 \mathrm{~mm} / \mathrm{s}
$$

$$
\mathbf{v}_{A}=\left[1061.3 \mathrm{~mm} / \mathrm{s} \triangle 32.005^{\circ}\right]
$$

$\operatorname{Rod} A B . \quad \mathbf{v}_{B}=v_{B} \longleftarrow$

$$
\sin \varphi=\frac{80}{250}, \quad \varphi=18.663^{\circ}
$$

Plane motion $=$ Translation with $A+$ Rotation about $A$.


Draw velocity vector diagram.

$$
\begin{aligned}
\delta & =180^{\circ}-\gamma-\left(90^{\circ}+\varphi\right) \\
& =90^{\circ}-32.005^{\circ}-18.663^{\circ}=39.332^{\circ}
\end{aligned}
$$

Law of sines.

$$
\begin{aligned}
\frac{v_{B}}{\sin \delta} & =\frac{v_{B / A}}{\sin \gamma}=\frac{v_{A}}{\sin \left(90^{\circ}+\varphi\right)} \\
v_{B} & =\frac{v_{A} \sin \delta}{\sin \left(90^{\circ}+\varphi\right)}=\frac{(1061.3) \sin 39.332^{\circ}}{\sin 108.663^{\circ}} \\
& =710 \mathrm{~mm} / \mathrm{s} \\
v_{B / A} & =\frac{v_{A} \sin \gamma}{\sin \left(90^{\circ}+\varphi\right)}=\frac{(1061.3) \sin 32.005^{\circ}}{\sin 108.663^{\circ}} \\
& =593.8 \mathrm{~mm} / \mathrm{s} \\
\omega_{A B} & =\frac{v_{B / A}}{\overline{A B}}=\frac{593.8}{250}=2.37 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$


$\mathbf{v}_{B}=710 \mathrm{~mm} / \mathrm{s} \longleftarrow$ -
$\boldsymbol{\omega}_{A B}=2.37 \mathrm{rad} / \mathrm{s}$ )
3. The $30-\mathrm{kg}$ roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 2.4 m for every one revolution, determine the speed of the car when $t=4 \mathrm{~s}$. Also, how far has the car descended in this time? Neglect friction and the size of the car.


$$
\begin{aligned}
& \theta=\tan ^{-1}\left(\frac{2.4}{2 \pi(2.4}\right)=9.043^{\circ} \\
& \Sigma F_{y}=0, N-30 \cdot 9.8 \cos 9.043^{\circ}=0 \\
& N=290.35 \mathrm{~N} \\
& H_{1}+\int M d t=H_{2} \\
& 0+\int_{0}^{4} 2.4\left(290.35 \sin 9.043^{\circ}\right) d t=30(2.4) v_{t} \\
& v_{t}=6.085^{\mathrm{m} / \mathrm{s}} \\
& v=\frac{6.085}{\cos 9.043^{\circ}}=6.161 \mathrm{~m} / \mathrm{s} \text { \# Ans. } \\
& T_{1}+\sum U_{1-2}=T_{2} \\
& 0+30 \times 9.8 \mathrm{~h}=\frac{1}{2} \cdot 30 \cdot 6.16^{2} \\
& \mathrm{~h}=1.936 \mathrm{~m} \text { \# Ans. }
\end{aligned}
$$


4. Collar $B$ moves to the left with a speed of $5 \mathrm{~m} / \mathrm{s}$, which is increasing at a constant rate of 1.5 $\mathrm{m} / \mathrm{s}^{2}$, relative to the hoop, while the hoop rotates with the angular velocity and angular acceleration shown. Determine the magnitudes of the velocity and acceleration of the collar at this instant.


Reference Frames: The $x y z$ rotating reference frame is attached to the hoop and coincides with the $X Y Z$ fixed reference frame at the instant considered, Fig. $a$. Thus, the motion of the $x y z$ frame with respect to the $X Y Z$ frame is

$$
v_{A}=a_{A}=\mathbf{0} \quad \omega=[-6 \mathbf{k}] \mathrm{rad} / \mathrm{s} \quad \dot{\omega}=\alpha=[-3 \mathbf{k}] \mathrm{rad} / \mathrm{s}^{2}
$$

For the motion of collar $B$ with respect to the $x y z$ frame,

$$
\begin{aligned}
& \mathbf{r}_{B / A}=[-0.45 \mathbf{j}] \mathrm{m} \\
& \left(v_{\mathrm{rel}}\right)_{x y z}=[-5 \mathbf{i}] \mathrm{m} / \mathrm{s}
\end{aligned}
$$

The normal components of $\left(\mathbf{a}_{\mathrm{rel}}\right)_{x y z}$ is $\left[\left(a_{\mathrm{rel}}\right)_{x y z}\right]_{n}=\frac{\left(v_{\mathrm{rel}}\right)_{x y z}{ }^{2}}{\rho}=\frac{5^{2}}{0.2}=125 \mathrm{~m} / \mathrm{s}^{2}$. Thus,

$$
\left(\mathbf{a}_{\mathrm{rel}}\right)_{x y z}=[-1.5 \mathbf{i}+125 \mathbf{j}] \mathrm{m} / \mathrm{s}
$$

Velocity: Applying the relative velocity equation,

$$
\begin{aligned}
\mathbf{v}_{B} & =\mathbf{v}_{A}+\omega \times \mathbf{r}_{B / A}+\left(\mathbf{v}_{\mathrm{rel}}\right)_{x y z} \\
& =\mathbf{0}+(-6 \mathbf{k}) \times(-0.45 \mathbf{j})+(-5 \mathbf{i}) \\
& =[-7.7 \mathbf{i}] \mathrm{m} / \mathrm{s}
\end{aligned}
$$

Thus,

$$
v_{B}=7.7 \mathrm{~m} / \mathrm{s} \leftarrow
$$

Ans.

Acceleration: Applying the relative acceleration equation,

$$
\begin{aligned}
\mathbf{a}_{B} & =\mathbf{a}_{A}+\dot{\omega} \times \mathbf{r}_{B / A}+\omega \times\left(\omega \times \mathbf{r}_{B / A}\right)+2 \omega \times\left(\mathbf{v}_{\mathrm{rel}}\right)_{x y z}+\left(\mathbf{a}_{\mathrm{rel}}\right)_{x y z} \\
& =\mathbf{0}+(-3 \mathbf{k}) \times(-0.45 \mathbf{j})+(-6 \mathbf{k}) \times[(-6 \mathbf{k}) \times(-0.45 \mathbf{j})]+2(-6 \mathbf{k}) \times(-5 \mathbf{i})+(-1.5 \mathbf{i}+125 \mathbf{j}) \\
& =[-2.85 \mathbf{i}+201.2 \mathbf{j}] \mathrm{m} / \mathrm{s}^{2}
\end{aligned}
$$

Thus, the magnitude of $\mathbf{a}_{B}$ is therefore

$$
a_{B}=\sqrt{2.85^{2}+201.2^{2}}=201 \mathrm{~m} / \mathrm{s}^{2}
$$

Ans.
5. In case of emergency, the gas actuator is used to move a $75-\mathrm{kg}$ block $B$ by exploding a charge $C$ near a pressurized cylinder of negligible mass. As a result of the explosion, the cylinder fractures and the released gas forces the front part of the cylinder, $A$, to move $B$ forward, giving it a speed of $200 \mathrm{~mm} / \mathrm{s}$ in 0.4 s . If the coefficient of kinetic friction between $B$ and the floor is $\mu_{k}=0.5$, determine the impulse that the actuator imparts to $B$.

$(\vec{~})$

$$
\begin{aligned}
& m\left(v_{x}\right)_{1}+\Sigma \int F_{x} d t-m\left(v_{x}\right)_{2} \\
& 0+\int F d t-(0.5)(9.81)(75)(0.4)-75(0.2) \\
& \int F d t-162 \mathrm{~N} \cdot \mathrm{~s}
\end{aligned}
$$

Ans
6. The disk rotates with the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link $A C$ at this instant. The peg at $B$ is fixed to the disk.

$\mathbf{v}_{B}=-6(0.3) \mathbf{i}=-1.8 \mathbf{i}$
$\mathbf{a}_{B}=-10(0.3) \mathbf{i}-(6)^{2}(0.3) \mathbf{j}=-3 \mathbf{i}-10.8 \mathbf{j}$
$\mathbf{v}_{B}=\mathbf{v}_{A}+\boldsymbol{\Omega} \times \mathbf{r}_{B / A}+\left(v_{B / A}\right)_{x y z}$
$-1.8 \mathbf{i}=0+\left(\omega_{A C} \mathbf{k}\right) \times(0.75 \mathbf{i})-\left(v_{B / A}\right)_{x y z} \mathbf{i}$
$-1.8 \mathbf{i}=-\left(v_{B / A}\right)_{x y z}$
$\left(v_{B / A}\right)_{x y z}=1.8 \mathrm{~m} / \mathrm{s}$
$0=\omega_{A C}(0.75)$
$\omega_{A C}=0$

$\mathbf{a}_{B}=\mathbf{a}_{A}+\dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B / A}+\boldsymbol{\Omega} \times\left(\boldsymbol{\Omega} \times \mathbf{r}_{B / A}\right)+2 \boldsymbol{\Omega} \times\left(v_{B / A}\right)_{x y z}+\left(\mathbf{a}_{B / A}\right)_{x y z}$
$-3 \mathbf{i}-10.8 \mathbf{j}=\mathbf{0}+\alpha_{A C} \mathbf{k} \times(0.75 \mathbf{i})+\mathbf{0}+\mathbf{0}-a_{A / B} \mathbf{i}$
$-3=-a_{A / B}$
$a_{A / B}=3 \mathrm{~m} / \mathrm{s}^{2}$
$-10.8=\alpha_{A / C}(0.75)$
$\alpha_{A / C}=14.4 \mathrm{rad} / \mathrm{s}^{2} 2$
Ans.

Ans.

