<u>注意：</u>

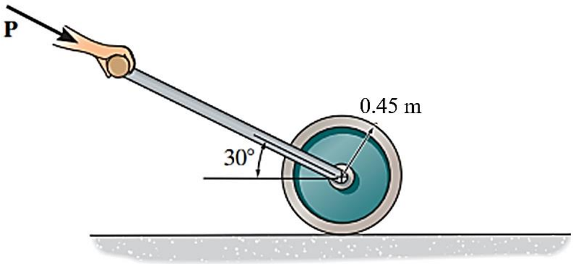
1.　所有題目均需詳列計過程，否則不予計分。

2.　除計算機外，禁止使用如書籍、筆記、講義等任何形式之輔助工具。

3.　請於試卷謄寫下列之 Honor Code。

“本人於此次考試中沒有接受任何形式之外部協助作答”(簽名)

1. If P = 130 N, determine the angular acceleration of the 22.5 kg roller. Assume the roller to be a uniform cylinder and that no slipping occurs.



$I_G = \frac{1}{2}mr^2 = \frac{1}{2}(22.5)(0.45)^2 = 2.278 \text{ kg}\cdot\text{m}^2$

$\xrightarrow{+}\Sigma F_x = m(a_G)_x;\quad 130\cos 30^\circ - F_f = 22.5\,a_G \qquad -(1)$

$+\uparrow\Sigma F_y = m(a_G)_y;\quad N - 22.5(9.81) - 130\sin 30^\circ = 0$

$N = 285.73\text{ N}$

$+\Sigma M_G = I_G\alpha;\quad F_f(0.45) = 2.278\,\alpha \qquad -(2)$

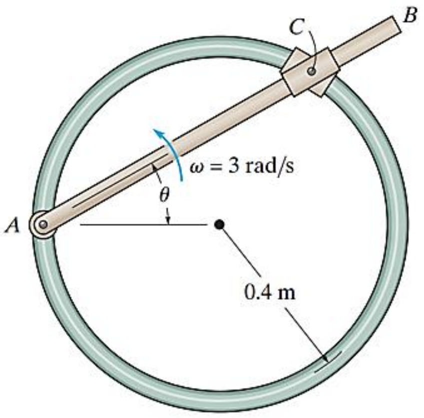
Since the roller rolls without slipping

$a_G = \alpha r = 0.45\alpha \qquad -(3)$

Solving (1) - (3)

$\Rightarrow \alpha = 7.413 \text{ rad/s}^2$ # Ans

2. Rod AB rotates counterclockwise with a constant angular velocity $\omega = 3$ rad/s. Determine the velocity and acceleration of point C located on the double collar when $\theta = 45°$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod AB.



SOLUTION

$\mathbf{r}_{C/A} = \{0.400\mathbf{i} + 0.400\mathbf{j}\}$

$\mathbf{v}_C = -v_C\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$-v_C\mathbf{i} = 0 + (3\mathbf{k}) \times (0.400\mathbf{i} + 0.400\mathbf{j}) + (v_{C/A}\cos 45°\mathbf{i} + v_{C/A}\sin 45°\mathbf{j})$

$-v_C\mathbf{i} = 0 - 1.20\mathbf{i} + 1.20\mathbf{j} + 0.707v_{C/A}\mathbf{i} + 0.707v_{C/A}\mathbf{j}$

$-v_C = -1.20 + 0.707v_{C/A}$

$0 = 1.20 + 0.707v_{C/A}$

$v_C = 2.40\text{ m/s}$ **Ans.**

$v_{C/A} = -1.697\text{ m/s}$

$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$$-(a_C)_t\mathbf{i} - \frac{(2.40)^2}{0.4}\mathbf{j} = 0 + 0 + 3\mathbf{k} \times [3\mathbf{k} \times (0.4\mathbf{i} + 0.4\mathbf{j})] + 2(3\mathbf{k}) \times [0.707(-1.697)\mathbf{i} + 0.707(-1.697)\mathbf{j}] + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$$

$-(a_C)_t\mathbf{i} - 14.40\mathbf{j} = 0 + 0 - 3.60\mathbf{i} - 3.60\mathbf{j} + 7.20\mathbf{i} - 7.20\mathbf{j} + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$

$-(a_C)_t = -3.60 + 7.20 + 0.707a_{C/A}$

$-14.40 = -3.60 - 7.20 + 0.707a_{C/A}$

$a_{C/A} = -5.09\text{ m/s}^2$

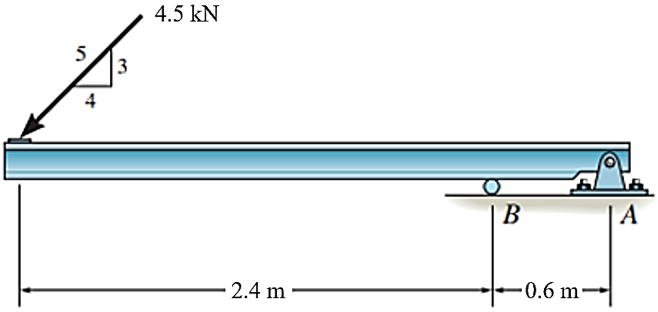
$(a_C)_t = 0$

Thus,

$$a_C = (a_C)_n = \frac{(2.40)^2}{0.4} = 14.4\text{ m/s}^2$$

$a_C = \{-14.4\mathbf{j}\}\text{ m/s}^2$ **Ans.**

3. The 230-kg beam is supported at *A* and *B* when it is subjected to a force of 4.5-kN as shown. If the pin support at *A* suddenly fails, determine the beam's initial angular acceleration and the force of the roller support on the beam. For the calculation, assume that the beam is a slender rod so that its thickness can be neglected.



$\overset{+}{\leftarrow} \sum F_x = m(a_G)_x$; $\quad 4500(\frac{4}{5}) = 230(a_G)_x$

$+\downarrow \sum F_y = m(a_G)_y$; $\quad 4500(\frac{3}{5}) + 230(9.81) - B_y = 230(a_G)_y$

$\circlearrowleft + \sum (M_k)_B$; $\quad 230(9.81)(0.9) + 4500(\frac{3}{5})(2.4) = 230(a_G)_y(0.9) + [\frac{1}{12}(230)(3)^2]\alpha$

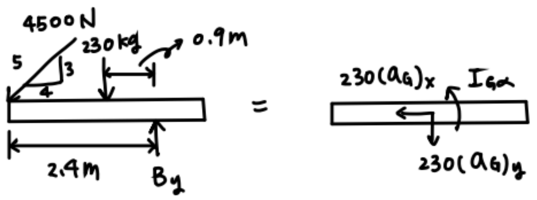
$a_B = a_G + a_{B/G}$

$-a_B\hat{i} = -(a_G)_x\hat{i} - (a_G)_y\hat{j} + \alpha(3)\hat{j}$

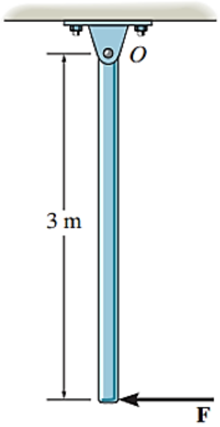
$(a_G)_y = \alpha(3)$

$\alpha = 23.71 \text{ rad/s}^2$ # Ans.

$B_y = 48.33$ N # Ans.



4. The 10-kg uniform slender rod is suspended at rest when the force of $F = 150$ N is applied to its end. Determine the angular velocity of the rod when it has rotated 90° clockwise from the position shown. The force is always perpendicular to the rod.



Kinetic Energy. Since the rod starts from rest, $T_1 = 0$. The mass moment of inertia of the rod about O *is* $I_0 = \frac{1}{12}(10)(3^2) + 10(1.5^2) = 30.0\ \text{kg}\cdot\text{m}^2$. Thus,

$$T_2 = \frac{1}{2}I_0\omega^2 = \frac{1}{2}(30.0)\,\omega^2 = 15.0\,\omega^2$$

Work. Referring to the FBD of the rod, Fig. a, when the rod undergoes an angular displacement θ, force **F** does positive work whereas **W** does negative work. When $\theta = 90°$, $S_W = 1.5$ m and $S_F = \theta r = \left(\frac{\pi}{2}\right)(3) = \frac{3\pi}{2}$ m. Thus

$$U_F = 150\left(\frac{3\pi}{2}\right) = 225\pi\ \text{J}$$

$$U_W = -10(9.81)(1.5) = -147.15\ \text{J}$$

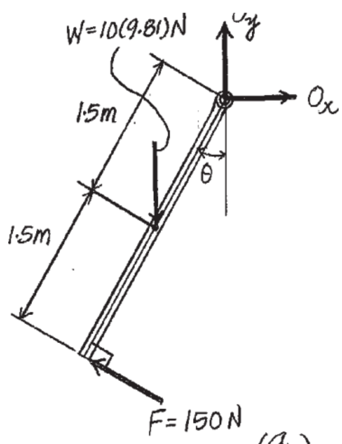
Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 225\pi + (-147.15) = 15.0\,\omega^2$$

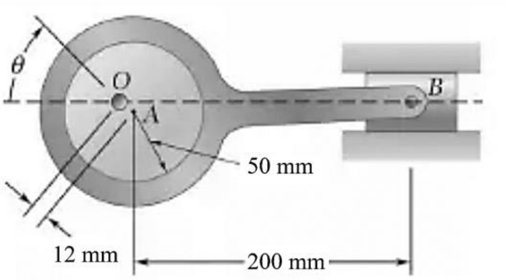
$$\omega = 6.1085\ \text{rad/s} = 6.11\ \text{rad/s}$$

Ans.

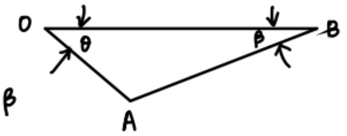


(a)

5. In the eccentric shown, a disk of 50-mm-radius revolves about shaft O that is located 12 mm from the center A of the disk. The distance between the center A of the disk and the pin at B is 200 mm. Knowing that the angular velocity of the disk is 900 rpm clockwise, determine the velocity of the block when $\theta = 30°$.



$(OA)\sin\theta = (AB)\sin\beta$



$\Rightarrow (0.012)\sin 30° = \dfrac{0.2}{\cos\beta}\sin\beta$

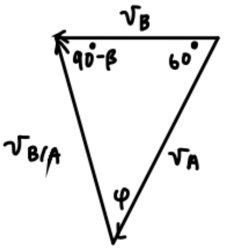
$\Rightarrow \tan\beta = 0.03$

$\Rightarrow \beta = 1.718°$

$\omega_{OA} = 900\ rpm = \dfrac{900}{60}(2\pi) = 94.25\ rad/s$

$v_A = (OA)\,\omega_{OA} = (0.012)\,94.25 = 1.131\ m/s \swarrow$

$v_B = v_A + v_{B/A} \quad [v_B \leftarrow] = [v_A \swarrow 60°] + [v_{A/B} \nwarrow \beta]$



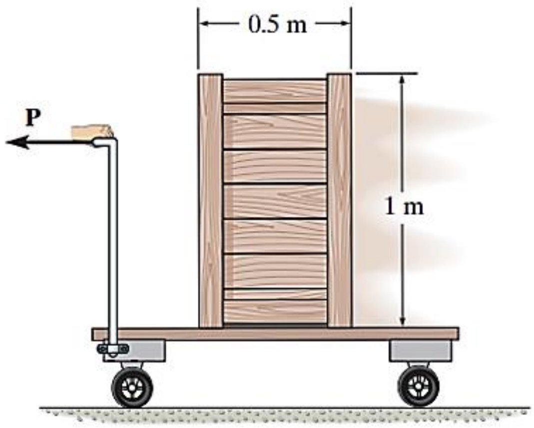
$90° - \beta = 88.28°$

$\varphi = 180° - (90° - \beta) - 60° = 31.72°$

$$\frac{v_B}{\sin\varphi} = \frac{v_A}{\sin(90° - \beta)}$$

$v_B = 0.594\ m/s$ # Ans.

6. The 150-kg uniform crate rests on the 10-kg cart. Determine the maximum force P that can be applied to the handle without causing the crate to slip or tip on the cart. The coefficient of static friction between the crate and cart is $\mu_s = 0.2$.



Equation of Motion. Assuming that the crate slips before it tips, then $F_f = \mu_s N = 0.2\,N$. Referring to the FBD and kinetic diagram of the crate, Fig. *a*

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 150\,(9.81) = 150\,(0) \qquad N = 1471.5\text{ N}$$

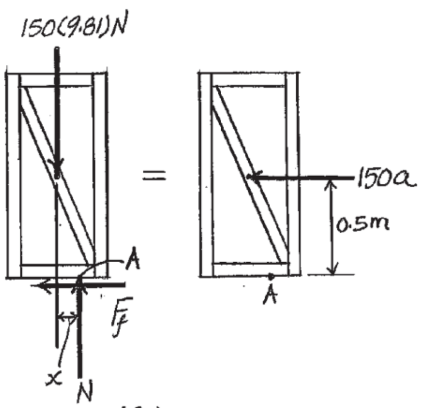
$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0.2(1471.5) = 150\,a \qquad a = 1.962\text{ m/s}^2$$

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 150(9.81)(x) = 150(1.962)(0.5)$$

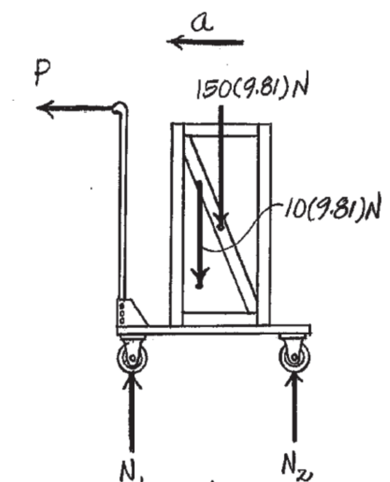
$$x = 0.1\text{ m}$$

Since $x = 0.1\text{ m} < 0.25\text{ m}$, the crate indeed slips before it tips. Using the result of a and refer to the FBD of the crate and cart, Fig. *b*,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P = (150 + 10)(1.962) = 313.92\text{ N} = 314\text{ N} \qquad \textbf{Ans.}$$



(a)



(b)